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Computations of unsteady transonic flow over airfoil at low Reynolds number

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Abstract: Transonic flow over a thin airfoil at low Reynolds number was studied numerically by directly solving two-dimensional full Navier-Stokes equations through 5th order weighted essentially non-oscillatory (WENO) scheme without using any turbulence model. A series of distinguished unsteady phenomena for a thin 2-D transonic airfoil flow were presented. Due to continuous adverse pressure gradient in the subsonic flow downstream of the sonic line, the unsteady separated boundary layer with main vortex and secondary vortex was developed at the rear of the airfoil. At the trailing edge, the vortex-shedding was characterized by periodical connection of the main vortex and secondary vortex on the other side of the airfoil. The unsteady separation and vortex-shedding occurred with the same period. On the airfoil surface, the average pulse pressure related to the unsteady supersonic region was obviously smaller than that related to the vortex-shedding at the trailing edge. With the attack angle increasing from 0° to 2° , the frequency of vortex-shedding decreases about 4.2%. At last, the turbulence intensity and many second-order statistics in the wake region were investigated.

Key words: transonic flow; unsteady flow; direct numerical simulation (DNS); unsteady separation; airfoil; vortex-shedding

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Transonic flow is characterized by mixed local subsonic and supersonic flow in the whole flow field. This complicates not only theoretical analysis but also experiments and numerical computations. However, transonic flow exists widely, such as in the wings of aircraft, in the tip of propellers, turbines and compressor blades and so on. In addition to these, there are many complex flow phenomena in the transonic flow field, for example, the unsteady separation boundary layer, vortex in the boundary layer, vortex-shedding at the trailing edge, shock and its interaction with boundary layer and vortex. Many unsteady phenomena, such as buffeting of the wings, surge of the jet engines, aeroelasticity and dynamic stall problems are associated

with the transient transonic flow. For the inviscid steady transonic flow, Ferrari and Tricomi^[1] have developed a relatively good mathematical and physical model; Jameson^[2] studied the non-unique solutions of Euler equations to solve the transonic flow over an airfoil; Cole and Cook^[3] studied the shock-foot singularity; Pearcey et al.^[4] made a rigorous research of separation after the shock and Lee^[5] studied the shock oscillations on the airfoil in the transonic flow. However, the physical mechanisms of the phenomena are still not fully understood, although many experiments have been performed during past several decades.

Because of the inherent nonlinearity of the governing equations, it is difficult to analyze the

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transonic flow theoretically even though the analysis is based on the small disturbances linear theory^[6]. Dimension analysis according to the similarity laws is another useful method to get the important non-dimensional parameters that must be considered in the analytical process. But in order to investigate the internal mechanism of the flow problems, it is obviously not sufficient at all.

With the development of the computer hardware, computational fluids dynamics (CFD) plays a more and more important role not only in solving the flow problems, but also in probing the flow rules of many complex flow^[7]. At present, Euler and Reynolds average Navier-Stokes equations (RANS) are widely used in mathematical models for fluids dynamics. But, both are not adequate to study the internal mechanism of unsteady separation, transition and many other complex flow problems. Although the full Navier-Stokes equations are ideal mathematical model, it is impossible to solve the equations numerically at high Reynolds number because of the unacceptable huge computational resources demanded. However, due to the advancement of computational algorithms and parallel computation, especially the high-order accuracy numerical methods, which significantly decrease computing cost, it is probable to make the direct numerical simulation (DNS) of low and middle Reynolds number flow reality^[7-8]. The unsteady flow field over a thin airfoil at low Reynolds number ($<10^5$) is studied mainly for theoretical attentions, because it contains many complex flow phenomena, such as unsteady separation, vortex-shedding, wakes and so on. Besides, it is important to investigate the mechanism of laminar boundary layer dynamic stall and leading edge separation^[9]. On the other side, the corresponding literature on the experimental study has not been found till now.

In this paper, the transonic flow at low Reynolds number around the airfoil NACA0012 was studied through directly solving the two-dimensional full Navier-Stokes equations numeri-

cally with 5th order weighted essentially non-oscillatory (WENO) scheme without using any turbulence model. The Reynolds number based on the chord is 10000, and the free stream Mach number is 0.8. The unsteady flow structure, separation boundary layer, and vortex-shedding at the trailing edge were investigated. Because the fully laminar flow can not be sustained above Reynolds number of 10 000, the turbulence intensity and many second order statistics in the wake region were also investigated^[9]. At last, the effect of the small angle of attack (AOA) at 1 and 2 degrees was also studied.

1 Computation scheme and meshes

The unsteady two-dimensional Navier-Stokes equations in general curve coordinates can be expressed as

$$\frac{\partial \hat{U}}{\partial \tau} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} = \frac{1}{Re} \left(\frac{\partial \hat{E}_v}{\partial \xi} + \frac{\partial \hat{F}_v}{\partial \eta} \right) \quad (1)$$

And the equations are dimensionless through ρ_∞ , T_∞ , V_∞ , L (chord length), and the pressure is dimensionless through $\rho_\infty V_\infty^2$. The Steger-Warming flux vector splitting technique is used for the inviscid terms on the left-hand side^[10]. This results in the positive flux term corresponding to the plus eigenvalues and negative flux term corresponds to the minus eigenvalues. Then the 5th order WENO^[11-13] scheme with limiters which maintain the upwind effect in both positive and negative flux terms is used to compute the convective terms. The viscous terms on the right-hand side are discretized using 6th order central finite difference scheme. At last, the 3th order accuracy total variation diminishing (TVD) Runge-Kutta method^[14] is used for time integration.

There is extra numerical dissipation in original WENO scheme which is disadvantage to obtain accurate results in the wake. Maximizing the bandwidth and adding limiters^[13] to the weight of the candidate stencils are usually used to mitigate the inherent numerical dissipation in original WENO scheme. And in this paper, the latter which is composed of absolute and relative limiters defined in Eq. (2) and Eq. (3) respec-

tively is used.

$$\omega_k = \begin{cases} C_k & \max(S_k) < A_{al} \\ \omega_k & \text{otherwise} \end{cases} \quad (2)$$

$$\omega_k = \begin{cases} C_k & \max(V_k)/\min(V_k) < A_{rl}^{tv} \\ & \text{and } \max(V_k) < B_{rl}^{tv} \\ \omega_k & \text{otherwise} \end{cases} \quad (3)$$

where ω_k is the weight of each candidate stencil, C_k is the optimal weight which introduces the least numerical dissipation; ω_k is the weight according to S_k which is the smoothness of the candidate stencil. The smoother the stencil is, the smaller the S_k is, and the bigger the ω_k is. The detail to compute the above parameters is given in Ref. [11]. V_k is the total variation on each candidate stencil; A_{al} is the threshold value used in absolute limiter, and A_{rl}^{tv} and B_{rl}^{tv} are the threshold values used in relative limiter. In Ref. [12], $A_{al} = 0.01$, $A_{rl}^{tv} = 5$ and $B_{rl}^{tv} = 0.2$ are adopted.

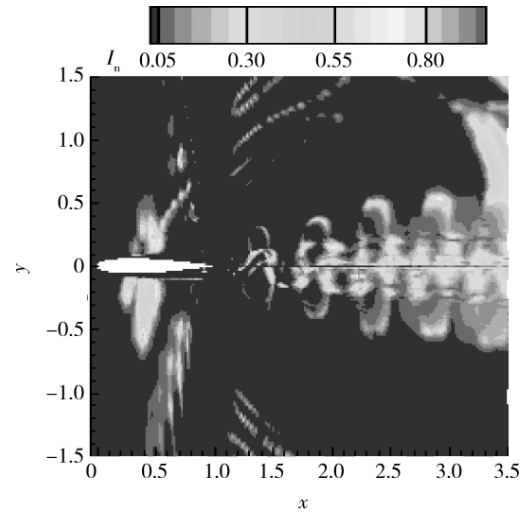
In order to measure the dissipation introduced by the WENO scheme, a nonlinear index I_n is defined^[12]

$$I_n = \frac{1}{\sqrt{r(r+1)}} \sqrt{\sum_{k=0}^r \left[1 - \frac{(r+1) \left(\frac{\omega_k}{C_k} \right)^2}{\sum_{k=0}^r \left(\frac{\omega_k}{C_k} \right)} \right]^2} \quad (4)$$

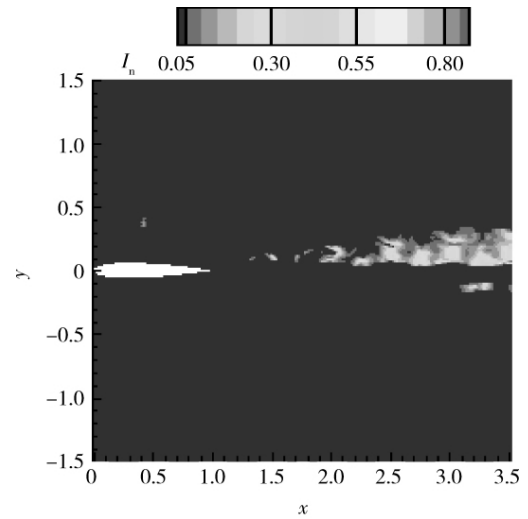
where r is the number of candidate stencils. I_n is a value between 0 and 1, whose magnitude indicates the dissipation added by the WENO scheme. In our numerical experiments, the relative limiter influences the stability of the computation, and the absolute limiter has more effects on the dissipation in the wake region. Fig. 1 shows the I_n distribution for different values of A_{al} and original WENO scheme in the wake region of more than two chords length with the same relative limiter defined above. As to the Steger-Warming splitting, the I_n value in the Fig. 1 is computed in the positive flux term with the biggest eigenvalue. As to the stability of computation, the $A_{al} = 0.05$ is used in the following computations.

Firstly, the validity of the numerical method for the transonic flow and the correction of

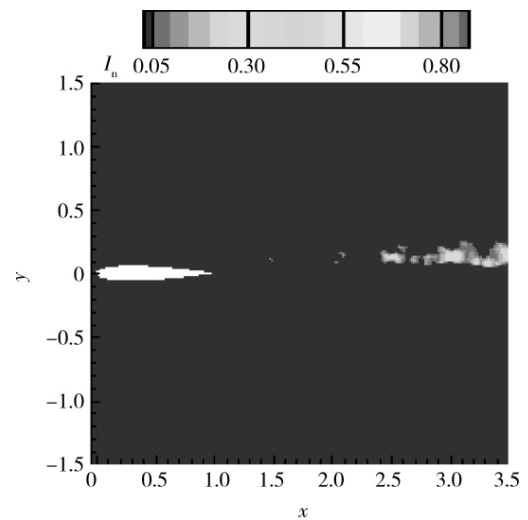
the code developed by this paper were examined by an inviscid transonic flow featured with the



(a) Original WENO



(b) $A_{al}=0.01$



(c) $A_{al}=0.05$

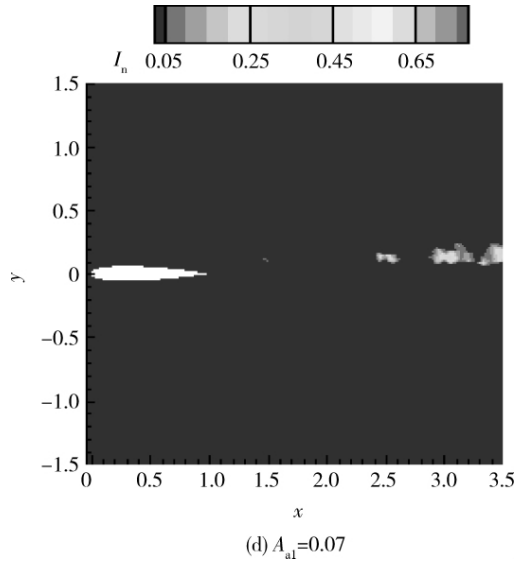


Fig. 1 I_n for original WENO and WENO added limiters with different A_{al}

free stream Mach number of 0.85 and the attack angle of 1° . Fig. 2(a) compares the computational result of this code with the experimental result from Advisory Group for Aerospace Research and Development (AGARD) of North Atlantic Treaty Organization^[15].

Then, for the viscous flows, body fitted C-type computational grids which are significantly stretched in normal direction and clustered near the trailing edge to respectively calculate the boundary layer and wake is used in these computations, and the grid boundary is located at 10 chords away from the airfoil in all directions. The WENO scheme used in this paper is 3th order accuracy at the discontinuous position and 5th order accuracy in the continuous area. In order to calculate the viscous term effectively, for the m th order accuracy method, the mesh space must be smaller than the critical space defined by the modified partial differential equations (MPDE) as follows^[16]:

$$\begin{aligned} \frac{\partial \hat{U}}{\partial \tau} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} = \\ \frac{(\Delta \epsilon)^\alpha}{Re(\Delta \epsilon)^\alpha} \frac{\partial \hat{E}_v}{\partial \xi} + \frac{(\Delta \eta)^\beta}{Re(\Delta \eta)^\beta} \frac{\partial \hat{F}_v}{\partial \eta} + \\ O[(\Delta \epsilon)^m, (\Delta \eta)^m] \end{aligned} \quad (5)$$

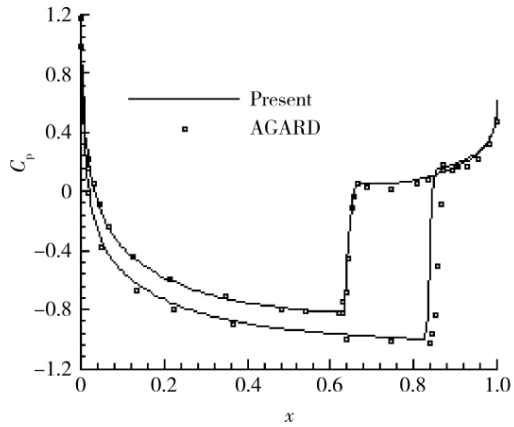
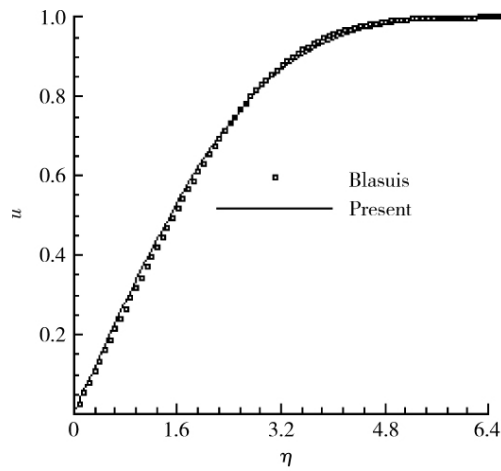
The critical mesh spaces satisfy

$$Re(\Delta \epsilon)^m = Re(\Delta \eta)^m = 1 \quad (6)$$

For $Re = 10\,000$, the critical mesh space is 0.046 for 3th order and 0.158 for 5th order accuracy method. In our computational mesh, the least mesh space is 0.00005; and at least within 7 chords length, the mesh space is not greater than 0.046. In the wake, the least mesh space from the trailing edge is 0.0005 in the flow direction and 0.00005 in the normal direction. The big mesh space near the outer boundary is advantage to eliminate the numerical dissipation reflection from outside of the computational area. The time step is about 0.00001 times of the vortex-shedding period. So, the mesh used in this paper is adequate for the direct numerical simulation and the numerical method is not only high order accuracy for the shock capture and whole transonic flow field but also with less numerical dissipation for the viscous calculation. In order to check the code reliability for viscous calculation, the laminar flat plate boundary layer was solved. And the comparison of the numerical solution and exact Blasius solution is shown in Fig. 2(b), where η is the non-dimensional boundary layer coordinate.

On the airfoil surface, the non-slip and adiabatic boundary conditions are used. For the far-away boundary and outflow boundary, the two dimensional Euler equations based on characteristic boundary conditions defined in Eq. (7) and Eq. (8) respectively are imposed, where $(\cdot)_b$ is the boundary value needed; $(\cdot)_e$ is defined through extrapolation on the boundary, and $(\cdot)_\infty$ is the free stream value. $\mathbf{n}$ in Eq. (7) and Eq. (8) is the inside normal direction of the boundary; $\mathbf{V}_b$ is the velocity vector, and u, v are the components in x and y directions respectively; ρ, p are the density and pressure, and c, γ are the sonic speed and adiabatic constant for the perfect gas.

$$\begin{cases} p_b = (p_\infty + p_e)/2 + \rho c (\mathbf{v}_\infty - \mathbf{v}_e) \cdot \mathbf{n} / 2 \\ \mathbf{v}_b \cdot \mathbf{n} = (p_\infty - p_e) / (2\rho c) + (\mathbf{v}_\infty + \mathbf{v}_e) \cdot \mathbf{n} / 2 \\ \rho_b = \rho_\infty (p_b / p_\infty)^{1/\gamma} \\ u_b = u_\infty + (\mathbf{v}_b \cdot \mathbf{n} - \mathbf{v}_\infty \cdot \mathbf{n}) n_x \\ v_b = v_\infty + (\mathbf{v}_b \cdot \mathbf{n} - \mathbf{v}_\infty \cdot \mathbf{n}) n_y \end{cases} \quad (7)$$

(a) C_p distribution along NACA0012 surfaces, inviscid flow

(b) Velocity profile of flat plate boundary layer

Fig. 2 Code verification examples

$$\begin{cases} p_b = (p_\infty + p_e)/2 + \rho c (\mathbf{v}_\infty - \mathbf{v}_e) \cdot \mathbf{n}/2 \\ \mathbf{v}_b \cdot \mathbf{n} = (p_\infty - p_e)/(2\rho c) + (\mathbf{v}_\infty + \mathbf{v}_e) \cdot \mathbf{n}/2 \\ \rho_b = \rho_\infty (p_b/p_e)^{1/\gamma} \\ u_b = u_e + (\mathbf{v}_b \cdot \mathbf{n} - \mathbf{v}_e \cdot \mathbf{n})n_x \\ v_b = v_e + (\mathbf{v}_b \cdot \mathbf{n} - \mathbf{v}_e \cdot \mathbf{n})n_y \end{cases} \quad (8)$$

Because of the periodicity of the solution, the initial values of the flow field are not very important, and the free stream condition is used for the initial values.

In order to get the mesh free results, the flow is computed on three meshes whose details are given in Table 1. In Table 1, N is the least number of points in the boundary layer, and the boundary layer edge speed is defined by the 0.995 of the free stream speed; f is the non-dimensional frequency of vortex-shedding at the trailing edge, which is defined as follow:

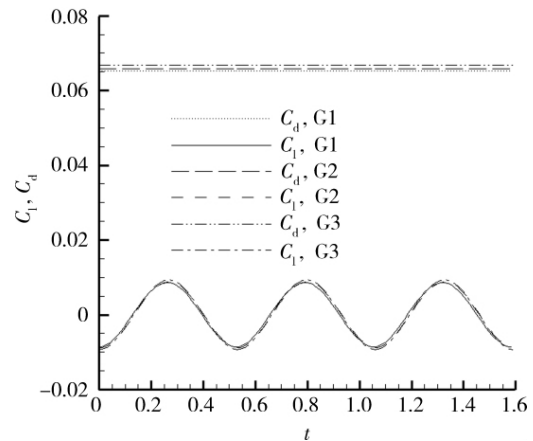
$$f = \tilde{f}L/V_\infty \quad (9)$$

where $\tilde{f}$ is the dimensional value of f .

To illustrate the mesh independence of the results, Fig. 3 shows the lift coefficient C_l and drag coefficient C_d of the airfoil at the attack angle of 0° during three periods on the three meshes. Unless especially stated, the following results are on the finest mesh G3.

Table 1 Grid parameters and vortex-shedding frequency

Grid	Dimension	N	f
G1	601×181	22	3.7948
G2	901×271	34	3.7152
G3	1201×361	45	3.7152

Fig. 3 C_d and C_l on three meshes, viscous flow

2 Results

2.1 Main structures of the flow

Fig. 4 and Fig. 5 illustrate the main structures of the low Reynolds number transonic flow over NACA0012 airfoil at the attack angle of 0° . As shown in Fig. 4(a), the main flow is mixed with supersonic and subsonic regions which are separated by the sonic line. The black solid line in Fig. 4(a) is the location of the boundary layer outer edge which is defined by the 0.995 of the free stream speed. It is obvious that the disturbances produced by the vortex-shedding and the wake are transferred upstream through subsonic flow in the boundary layer and main subsonic flow out of the boundary layer. Both contribute to the instantaneity of the supersonic region upstream which is the well-known circulation in the

transonic flow. Fig. 4(b) shows the vortices in the wake region which are produced by the periodic vortex-shedding at the trailing edge displayed in Fig. 5. In order to show the vortices structure clearly, it is necessary to minus the velocity u with the convective velocity of the vortices. The vortices structures shown in Fig. 4(b) are obtained by subtracting $0.85U_\infty$ from u velocity. The similar result is obtained with the experiments performed in Ref. [17] and computation in Ref. [18].

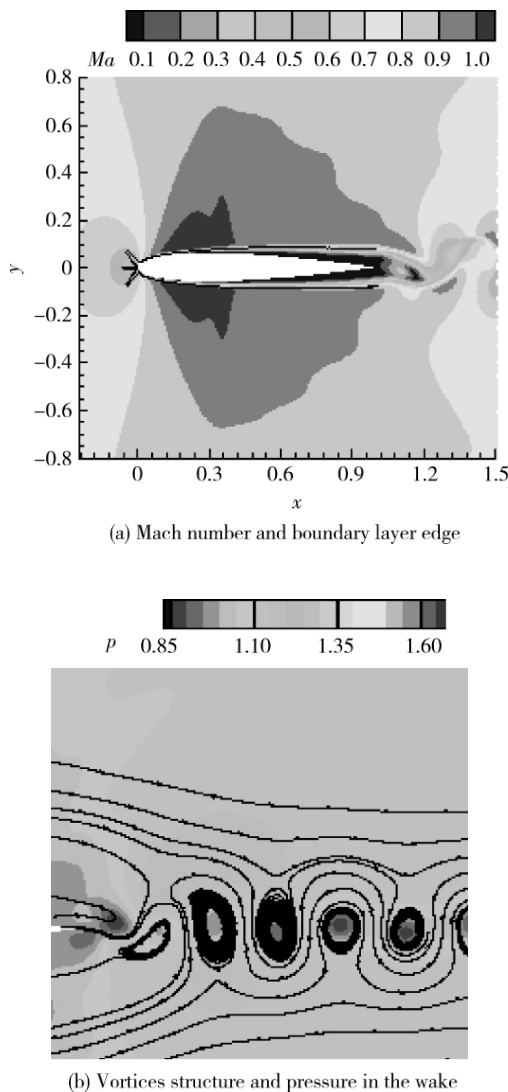


Fig. 4 Main structures of the flow

Due to the low Reynolds number in our computation, the sonic line is at about 41% chord and the maximum Mach number in the supersonic region is 1.08 which is less than the Mach number to generate separation on the air-

foil surface^[5]. Because the existing shock is very weak, the unsteady separated boundary layer is formed by the continuous adverse pressure gradient in the subsonic flow downstream of the sonic line. This is the so called B-model separation according to Pearcey et al.^[4]. Fig. 5 shows the process of vortex-shedding on the up airfoil surface. As depicted in Fig. 5, there are primary and secondary vortices in the separated boundary layer, and a periodical vortex-shedding dominates the wake structure at the trailing edge.

2.2 Vortex-shedding

Fig. 5 shows the flow field near the trailing edge at the times $0, T/4, 3T/8, T/2$, where T is the period of vortex-shedding at the trailing edge and Fig. 6 shows the pressure coefficient distribution on the airfoil surface at the same time. In order to show clearly, only rear 0.6 chord length is shown in Fig. 6. In Fig. 5(a), the vortex V1 which rotates anticlockwise has just left the lower airfoil surface. The interaction between vortex V1 and the high speed main flow out of boundary layer makes the pressure near point P_1 higher than the pressure near point P_2 . As a result, at the rear of the airfoil, the pressure on the upper airfoil becomes higher than that on the lower airfoil which is strongly affected by the low pressure kernel of vortex V1. At the same time, the high pressure in the area P_1 sustains the adverse pressure gradient on the upper surface near the trailing edge, which is necessary to form the vortex V2 rotating clockwise on the up airfoil surface. As shown in Fig. 6(a), because of the low pressure at the trailing edge on the lower airfoil, the stream line on the upper airfoil is round the trailing edge into the flow field of lower airfoil, which makes a secondary vortex rotating clockwise too on the lower surface and a separation saddle point on the upper airfoil surface.

In Fig. 5(b), as the vortex V1 moves downstream and the vortex V2 on the upper airfoil gets close to the trailing edge, the pressure at point P_1 becomes low. Because the vortex V1 gets away from the trailing edge, the pressure at

point P_2 becomes high, which provides the adverse pressure gradient needed to form a vortex. As a result, the vortex V4 which rotates anti-clockwise exists on the lower airfoil surface. As shown in Fig. 6(b), the pressures at trailing edge on upper and lower surfaces are the same,

which makes the separation saddle point on the upper airfoil surface disappeared. And near the trailing edge, the pressure on the low surface possesses relatively big adverse gradient. Consequently, during this time, the secondary vortex V3 on the lower airfoil becomes bigger and bigger.

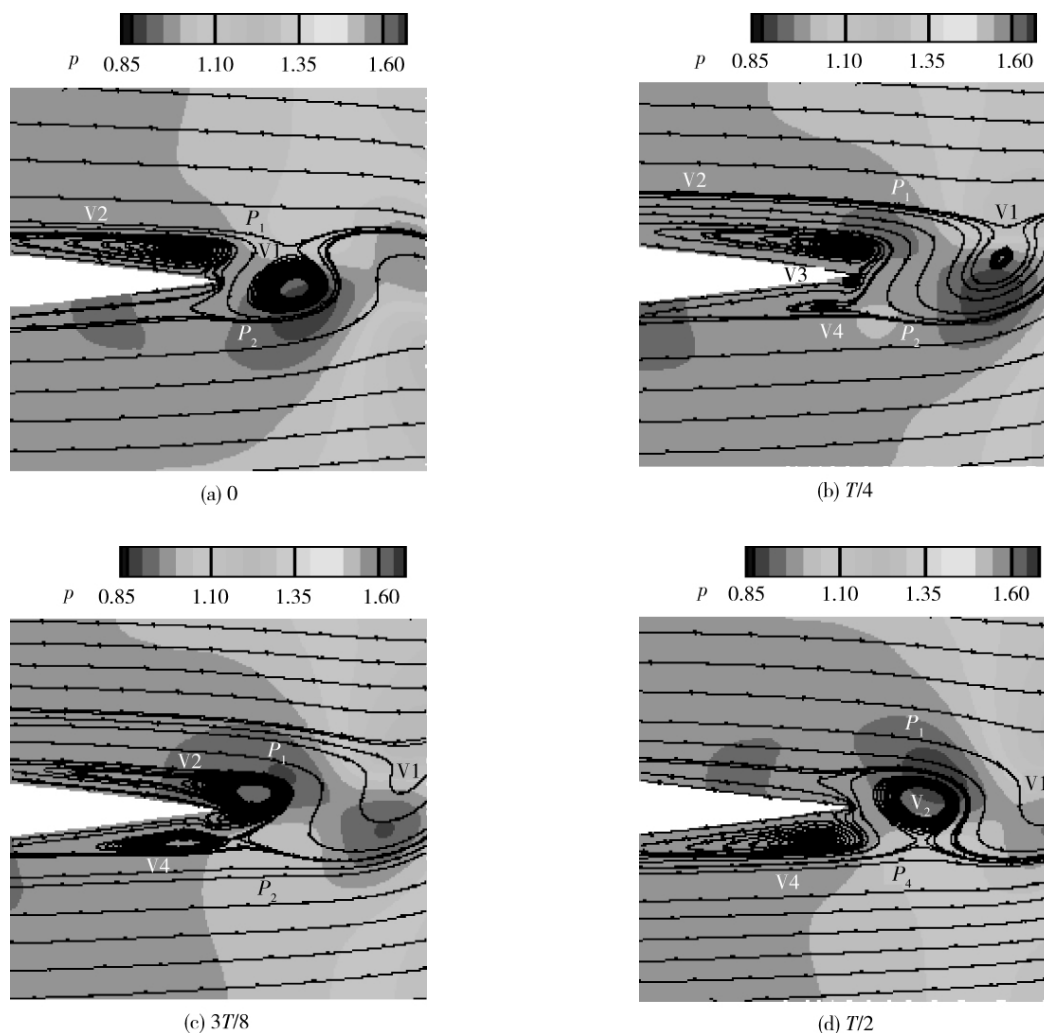


Fig. 5 Vortex-shedding at the trailing edge

In Fig. 5(c), the vortex V1 has merged into the wake. The vortex V2 on the upper surface in the separated boundary layer moves downstream and will leave the trailing edge, which makes the pressure at point P_2 much higher and that at point P_1 much lower. As to the high pressure at the point P_2 , relatively big adverse pressure gradient exists near the trailing edge of the lower surface, which makes the vortex V4 stronger. In Fig. 6(c), it is obvious that the pressure on lower surface is higher than that on upper surface in the rear of the airfoil. This results in the sec-

ondary vortex on the lower airfoil becomes open, and connects with the vortex V2 on the upper airfoil, which makes a separated saddle point on the lower surface.

In Fig. 5(d), the clockwise rotated vortex V2 in the upper separated boundary layer has left the trailing edge, which makes the pressure near the point P_1 lower than the pressure near the point P_2 . Periodical vortex-shedding makes the staggered pressure near the trailing edge. During this time, an adverse pressure gradient exists near the lower airfoil trailing edge, which

makes the vortex V4 bigger. As shown in Fig. 6 (d), the pressure on lower surface is still higher than that on upper surface in the rear of airfoil. As a result, the stream line on the lower airfoil is round the trailing edge into the upper flow field, which makes a secondary vortex rotating anticlockwise just as the vortex V4 on the upper surface. And a separation saddle point still exists on the lower airfoil surface. At that time, the flow structure on the upper and lower surfaces are the same as the lower and upper surfaces respectively at the beginning time. The vortex V4 on the lower airfoil will shed in the same way as the vortex V2 and periodical vortex-shedding forms the periodical staggered pressure and vortices street as shown in Fig. 4(b).

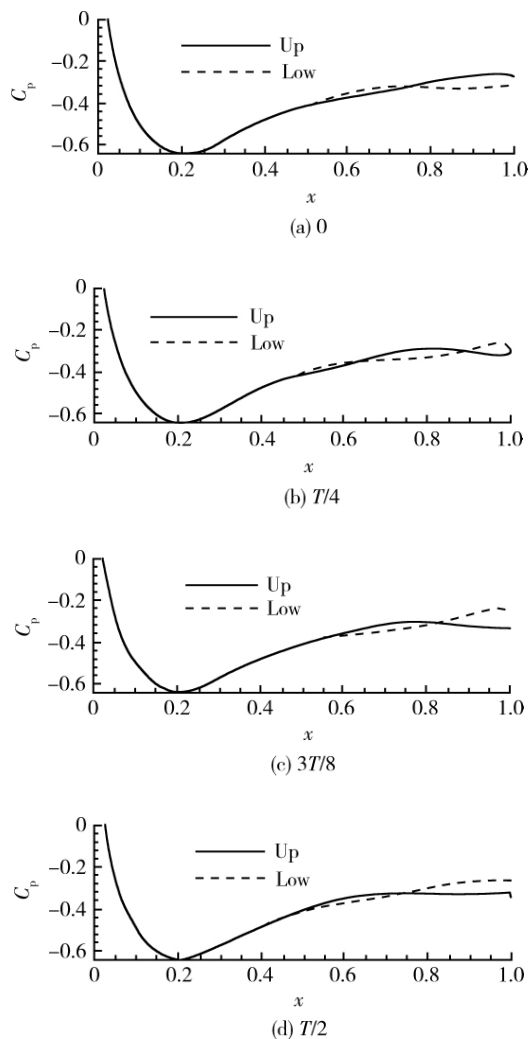


Fig. 6 Pressure coefficient on the airfoil

The staggered pressure plays an important

role during the vortex formation. Fig. 7(a) and Fig. 7(b) show the experimental results of vortex formation carried in Ref. [19], in which Reynolds number is 2.39×10^5 and attack angle is 4° . Fig. 7(a') and Fig. 7(b') show the computational results. In Fig. 7(a'), the anticlockwise vortex V1 produces a high pressure area near point P_1 , and the new forming vortex V2 produces another high pressure area near point P_2 . The staggered high pressure areas near the points P_1 and P_2 enhance the entrainment of the vortex V2 and promote the growth of vortex V2. The similar mechanism can be found in the experimental results correspondingly.

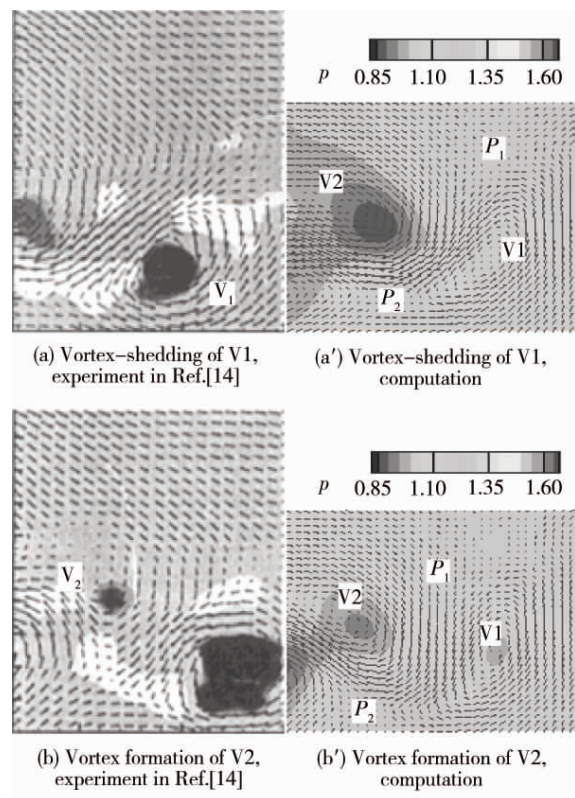


Fig. 7 Comparison of experimental and computational vortex formations near NACA 0012 trailing edge

2.3 Unsteady separation

As to the B-model separation, the adverse pressure gradient behind the sonic line induces the rear separation of the airfoil. The periodical vortex-shedding results in the periodical movement of the separation point. The location of sonic line is unsteady too, but the magnitude of

its movement is very small.

According to Moor et al., the separation points on the airfoil should satisfy

$$u_s = \frac{\partial u_s}{\partial n} = 0 \quad (10)$$

where s and n are the tangential and normal directions of the airfoil respectively and u_s is the tangential velocity. Because of the non-slip boundary condition on the airfoil, $u_s=0$ is satisfied obviously. This means that the zero point of the skin friction line is just the location of the separation point.

Fig. 8 shows the skin friction coefficients of the upper and lower airfoil surfaces at the times $0, T/4, T/2, 3T/4$. In order to show clearly, only rear 0.6 chord length is shown. In Fig. 8(a), the separation points on upper and lower airfoil surfaces are almost at the same location. Owing to the effect of vortices in the separated boundary layer, the separation point on the upper airfoil surface moves forward, while on the lower airfoil surface, the separation point moves backward at the same time. In Fig. 8(b), the

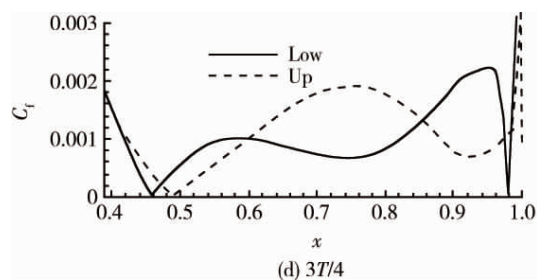
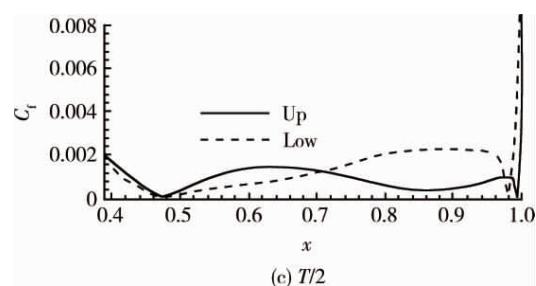
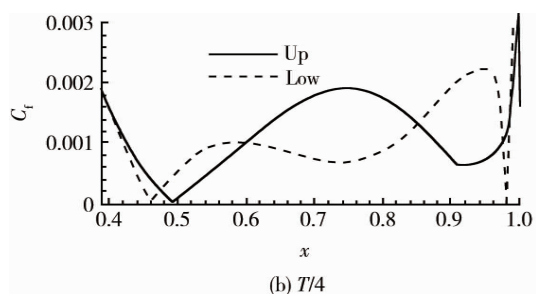
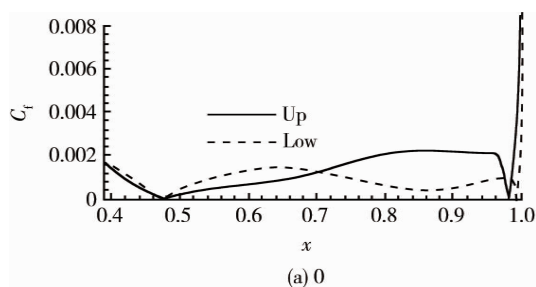
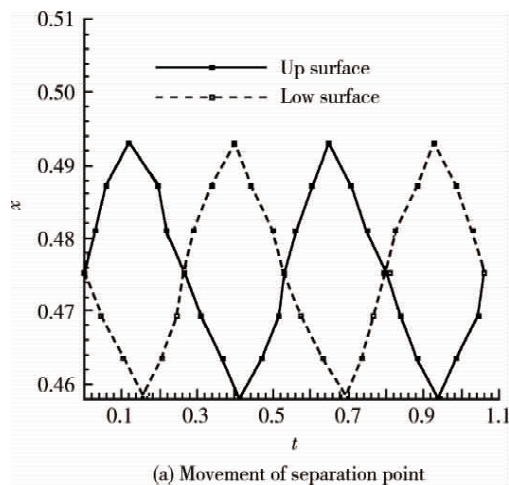


Fig. 8 Coefficients of skin friction on the airfoil

upper separation point is at almost the most forward position, and the lower one is at almost the most backward position. In Fig. 8(c), the separation points on upper and lower airfoil surfaces are almost at the same location too. But, the distribution of skin friction coefficient on the upper and lower airfoil surfaces at this time is the same as the distribution on the lower and upper surfaces in Fig. 8(a) respectively. The same relationship exists between Fig. 8(d) and Fig. 8(b). Fig. 9(a) shows the movement of separation point on upper and lower surfaces during two vortex-shedding cycles.

As to the periodic vortex-shedding, the pressure near the trailing edge changes periodically with almost the same period. Fig. 9(b) shows the pressure change at the first point behind the trailing edge in the wake center line, and the distance between this point and trailing edge is 0.0005 chord. As shown in Fig. 9, the unsteady separation points on the upper and lower airfoil surfaces move periodically with same period as the vortex-shedding and repeatedly with nearly the same $x(t)$ trace.



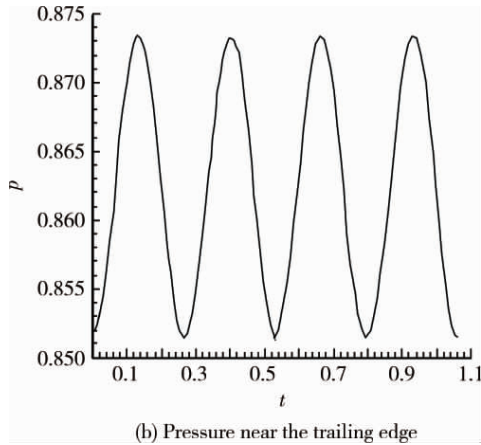


Fig. 9 Separation movement and pressure variation at attack angle of 0°

2.4 Statistic of wake

The turbulence already exists in wake at Reynolds number of 10 000. This subsection investigates the distribution of turbulence intensity and many second order statistics in the wake region. The turbulence intensity T_t is defined as

$$T_t = \frac{\sqrt{(1/2)(\overline{u'^2} + \overline{v'^2})}}{U_\infty} \quad (11)$$

where u' and v' are the pulse velocity in x , y directions; U_∞ is the free stream velocity, and the average is calculated in a period of vortex-shedding at the trailing edge. Fig. 10 shows the distribution of the turbulence intensity and the second order statistics $\overline{u'u'}$, $\overline{v'v'}$ and $\overline{u'v'}$ in the wake region at attack angle of 0° .

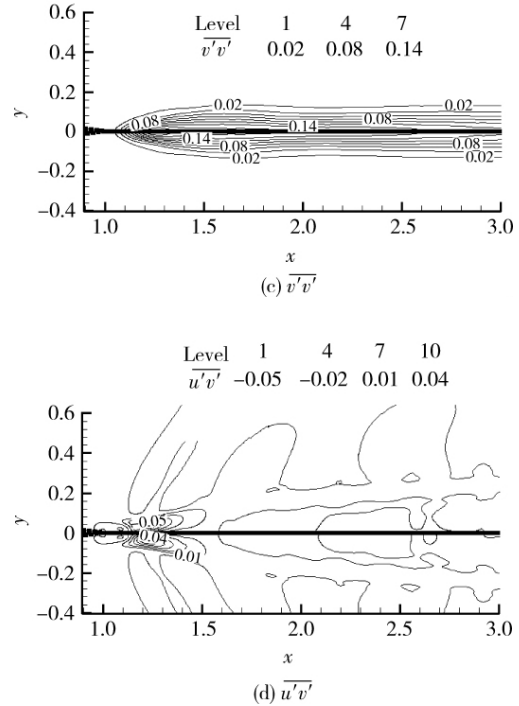
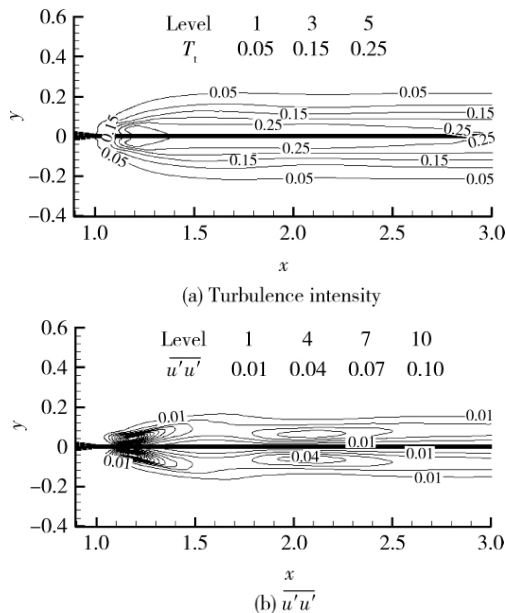


Fig. 10 Statistics of wake at attack angle of 0°

The upper and lower turbulence intensity is symmetric. The intensity increases, then decreases in the flow direction and the wake extends more than 8 times of the chord. In the normal direction, the maximum is in the center of the wake. The structure of $\overline{v'v'}$ is similar to the turbulence intensity T_t . The $\overline{u'u'}$ is symmetric about the center line of wake, which increases firstly, then decreases in the flow direction.

But, in the normal direction, there are two $\overline{u'u'}$ maximums which are symmetric on the center line. The distribution of $\overline{u'v'}$ is a typical double tails structure and is antisymmetric about the centre line of the wake. The magnitude of $\overline{u'v'}$ increases firstly, then decreases. The sign of $\overline{u'v'}$ is alternative in the flow direction and normal direction. These results coincide with the experimental results in Ref. [17] and the computational results in Ref. [18].

2.5 Influence of small attack angle

The AOA influences the statistics distribution in the wake. Fig. 11 shows the turbulence intensity, $\overline{u'u'}$, $\overline{v'v'}$ and $\overline{u'v'}$ distributions in the wake at attack angle of 2° . It is obvious that the wake center line is not the symmetric line of sta-

tistics. Actually, in Fig. 11(a), the maximum of the turbulence intensity is a little bigger than that in Fig. 10(a). In Fig. 11(b) and Fig. 11(d), the maximal magnitude of lower part is smaller than upper part. This coincides with the computational results of Ref. [18].

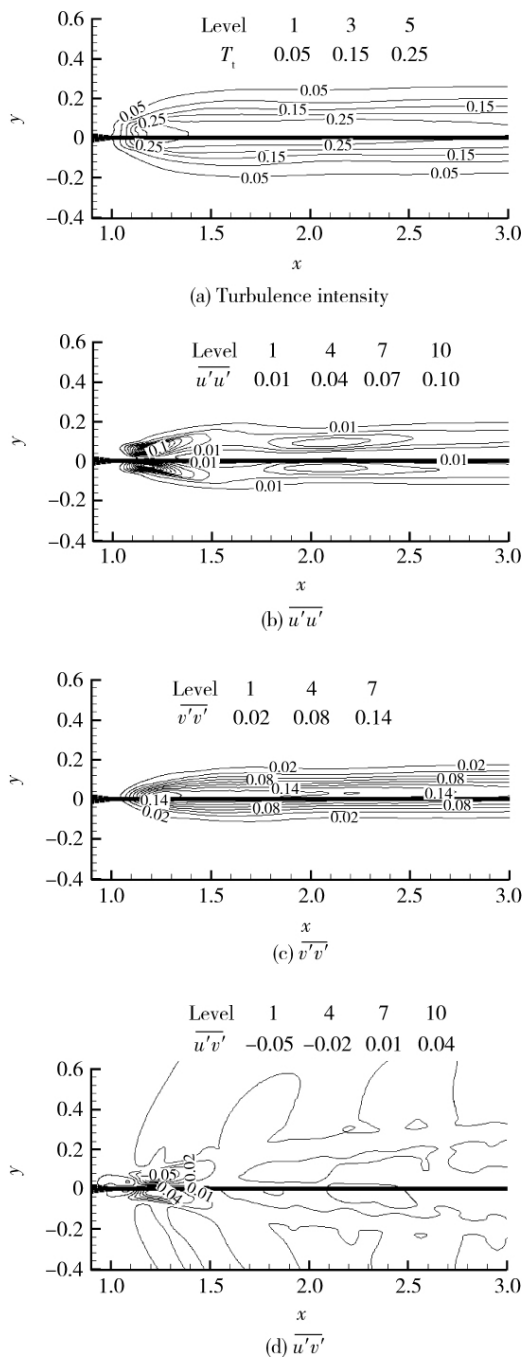
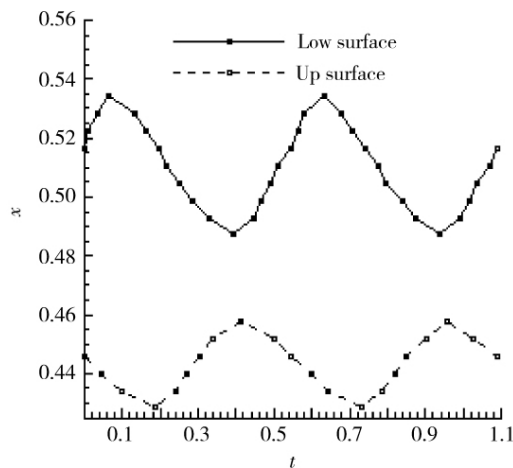


Fig. 11 Statistics of wake at attack angle of 2°

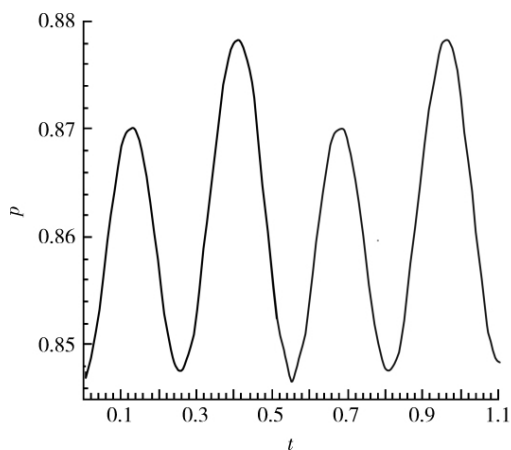
Fig. 12 shows the unsteady separation and pressure variation near the trailing edge at attack angles of 1° and 2° . The pressure sampling point is the first mesh point behind the trailing edge in

the wake central line, and the distance between the point and trailing edge is 0.0005 chord. It is obvious that the period of unsteady separation is the same as the vortex-shedding at small attack angle.

In Fig. 12(b), the location of separation on lower surface is at the rear of the airfoil. And the unsteady separation is affected by the secondary vortex as the vortex V3 in Fig. 5(b). When the secondary vortex connects with the primary vortex, the separation point on the lower surface moves backward rapidly. As the primary vortex like the vortex V4 in Fig. 5(c) gets stronger, the adverse pressure gradient makes the separation point move forward. Table 2 shows the frequency of vortex-shedding at the trailing edge and the average separation positions on the up and low airfoil surfaces at different attack angles. It is obvious that frequency of vortex-shedding decreases as the attack angle increases.



(a) Movement of separation point at attack angle of 1°



(a') Pressure near trailing edge at attack angle of 1°

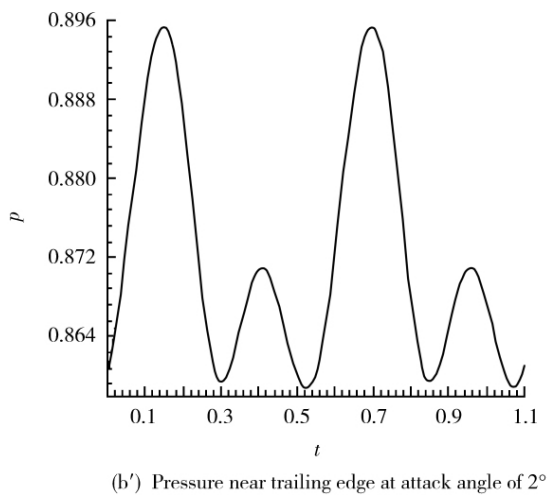
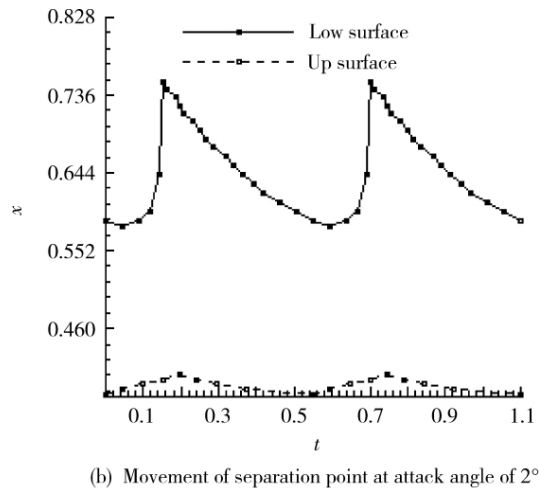


Fig. 12 Separation movement and pressure variation at attack angles of 1° and 2°

Table 2 Vortex-shedding frequency and average separation positions at different AOA

$A_{\text{on}} / (^\circ)$	0	1	2
f	3.715 18	3.583 75	3.557 60
Up surface	0.475 23	0.445 75	0.387 86
Low surface	0.475 23	0.504 81	0.620 38

Fig. 13 shows the average pulse pressure in the airfoil surface at different attack angles. Both the unsteady of supersonic region ahead of sonic line and vortex-shedding at the trailing edge produce the average pulse pressure on the airfoil. As shown in Fig. 13, the bigger the attack angle is, the stronger the average pulse pressure on the front of airfoil is. But, with small attack angle, the average pulse pressure

produced by the vortex-shedding at trailing edge is stronger and wider in the whole flow field than that produced by the moving of supersonic region.

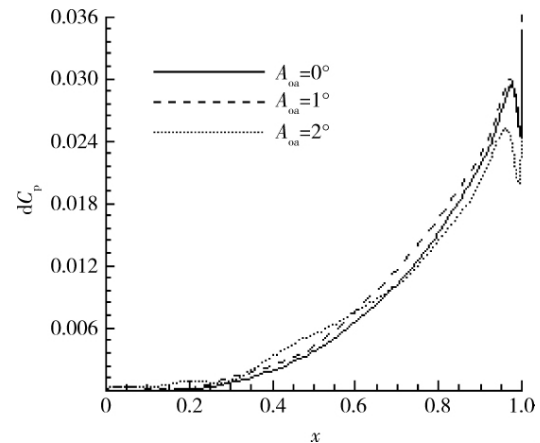


Fig. 13 Average pulse pressure on the airfoil at different attack angles

3 Conclusions

1) As to the relatively low Mach number of free stream and small attack angle, the shock is so weak that it can not induce the separation on the thin airfoil at low Reynolds number. However, the continuous adverse pressure gradient existing behind the supersonic region is the main reason for the rear separation of the thin airfoil.

2) In the separated boundary layer, there are a primary vortex, a secondary vortex and a separation saddle point on each surface of a thin airfoil. The vortex-shedding is characterized by periodic connection between the primary and secondary vortices.

3) The period of vortex-shedding at the trailing edge is the same as that of the movement of separation points on airfoil surfaces. And, with the increasing of attack angle, the frequency of vortex-shedding decreases.

4) At small attack angle, the statistics in the wake tend to lose its symmetric about the wake center line. The bigger the attack angle is, the stronger the average pulse pressure caused by the unsteady supersonic region is. But at small attack angle, the average pulse pressure produced by the vortex-shedding is much stron-

ger than that produced by the unsteady supersonic region.

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